Article title: Mean-Variance model: Added value of family business stocks.

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Abstract

Family business performance has been widely explored in the literature from corporate perspective. An open question is how individual investors can benefit from this performance in return-risk portfolio investment setting. This paper investigates the benefit of diversification using family business stocks in financial portfolios. Based on the geometric representation of the minimum variance frontier and asymptotic spanning tests, this paper highlights the important role of family business stocks in risk contraction in diversified financial portfolios.

Keywords

family business, mean-variance model, spanning test, step-down procedure, diversification.

1. Introduction

For decades, a burgeoning literature debates what could be a specific for a large part of organizational firms in the world; family businesses. This question seems to be relevant since family business is the oldest and most common model of economic organization. The interest of scholar works in family business is motivated, among others, by the impact and the
performance of this organization globally. According to (Sanker and Astrachan 1996), family businesses account for over 80% of all firms, 12% of GDP, and 15% of the workforce in the United States. Large part of family businesses are small, they represent more than 35% of the companies listed on the Standard & Poor (S&P) 500 or the Fortune 500 Index (Anderson and Reeb 2003).

From performance perspective, family business draws a particular attention. Large body of literature investigates the performance of family businesses and tries to contrast it with non-family firms. The focus of most of these efforts has been carried out to understand the relationship between the ownership structure, family involvement in business and other family business attributes with the performance of the firm. In this line, accounting and market variables have been extensively utilized through regression based models with a bench of specification. (Lindow et al. 2010; Maury 2006; Sraer and Thesmar 2007) used Return On Assets (ROA); Ebit; Ebitda; Return On Equity (ROE) as accounting variables, Return On Sales (ROS) by (Cucculelli and Micucci 2008; Graves and Shan 2013) Tobin’s \( q \) is used to measure the market performance by (Anderson and Reeb 2003; Chu 2009; Cronqvist and Nilsson 2003; Villalonga and Amit 2006).

Numerous scholarly works try to shed light on the performance of family business over their counterparts using different specification regression based model, however we are not aware of any contribution translating this performance from corporate setting to investment opportunity in the stock market for individual investors. Do family business stocks have an impact on stock portfolios in the return-risk dimensions? Can an individual investor in the stock market get higher and/or lower risk when diversifying his or her financial portfolio over family business stocks?

The present contribution investigates the added value of portfolio diversification using Family Business (FB) stocks in the return-risk dimensions. Findings are based upon contrasted
geometric representation of markowitz minimum efficient frontiers using family and Non Family Business (NFB) stocks. The visual results are supplemented by asymptotic regression based tests known in literature as spanning tests.

To investigate the well accepted statement of family business resilience to crisis period (Lemmon and Lins 2003; Chrisman et al. 2011), results are reported for two time frames: During financial crisis 2008 and post crisis period using two market measures: Portfolio expected returns and risks in morocco stock market.

The remainder of the paper is organized as follows, section 2 presents the mean-variance portfolio foundations serving as framework for gauging the return and risk, section 3 develops the asymptotic spanning tests, section 4 presents the empirical illustration along with results and section 5 concludes the paper.

2. Mean-Variance Model (MV)

The major advance in portfolio theory has been the recognition of interaction between assets and shows that the creation of an optimum investment portfolio relies on the ability of diversifying and combining several individual assets (Markowitz 1952). The findings of Markowitz established a widespread acceptance from academic and professional communities.

Definitions. We first start by developing some basic definitions. Let’s consider the problem of composing a portfolio of N assets. This can be represented by a weight vector \( x = (x_1, x_2, \ldots, x_N) \) with a sum constraint of weights equal to \( \sum_{i=1}^{N} x_i = 1 \). We assume that short selling is excluded, meaning that all weights are positive numbers, \( x_i \geq 0 \) for all \( i \in \{1,2,\ldots,N\} \).

Thus, portfolios universe is represented by:

\[
\mathcal{X} = \left\{ x \in \mathbb{R}^N, \sum_{i=1}^{N} x_i = 1, x \geq 0 \right\}
\]

Assets are characterized by an expected return \( E[R_i] \) for \( i \in \{1,2,\ldots,N\} \), and by variance-covariance matrix \( \Omega \) as a measure of risk with:
\[ \Omega_{ij} = \text{Cov}[R_i, R_j] = E[(R_i - E[R_i])(R_j - E[R_j])] \text{for } i, j \in \{1, \ldots, N\} \]

The expected return of portfolio \( x \) and its variance are defined as follow:

\[ E[R(x)] = \sum_{i=1}^{N} x_i E[R_i] ; \text{Var}(R(x)) = E[(R(x) - E[R(x)])^2] = \sum_{i,j=1}^{N} x_i x_j \Omega_{ij} \]

The quadratic program (P1) leads to an optimal portfolio:

\[
\begin{align*}
\text{Min} \ & V(R(x)) = \sum_{i,j=1}^{N} x_i x_j \Omega_{ij} \\
\text{s.c} \ & E(R(x)) = E^* \\
\sum_{i=1}^{N} x_i = 1 \text{ and } x_i \geq 0 \text{ for } i \in \{1, \ldots, N\}
\end{align*}
\]

Under the first constraint \( E^* \) presents a fixed value of return desired by the investor. When short selling is permitted, the quadratic program can be soothed through removing the non-negativity constraint.

The quadratic program, if feasible solution exists, leads to an optimal portfolio with a contraction of the risk given the desired return level. By varying the expected return for a range of values, one obtains all optimal portfolios in the MV dimensions from which the subset that are not strictly dominated in one or the other dimension by other portfolios. This is called the minimum variance efficient frontier.

While the above quadratic program looks for risk contraction given a certain level of return, (Briec et al. 2004)\(^1\) introduces a general framework deriving a performance measure, seeking improvement in both directions i.e., increasing return and decreasing risk.

In the current analysis, the program P1 is employed to derive efficient portfolios within the MV model. Geometrically, efficient frontiers are generated using several levels of returns and several panel assets. The representation of minimum variance frontiers allows a geometric

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\(^1\) (Briec et al. 2004) transposes the shortage function as a measure of efficiency from production theory to portfolio theory. The shortage function measures the distance between the asset under-evaluation and the efficient frontier in the direction of return improvement and risk contraction.
interpretation of the prevalence of an asset class over another and hence shows the benefit of diversification.

Furthermore, the geometric representations are supplemented by spanning statistical tests widely applied to gauge the benefit of diversification. The mean–variance spanning test\(^2\), initially introduced by (Huberman and Kandel 1987), tests the impact of including N risky assets. It is referred to as test assets and has on the minimum variance efficient frontier of an investment opportunity set of K called benchmark assets. If the minimum variance efficient frontiers derived from test assets N and all universe assets N+K coincide in the Mean Variance space, we have the case of spanning i.e., there is no statistical significance of an added value in both dimensions Mean and Variance of diversification over test assets. Investor can hold only optimal portfolio derived from Benchmark assets K and the inclusion of any additional asset from N will not yield to higher return or lower risk.

3. Spanning Tests
3.1 Joint-tests

The diversification benefits of FB stocks are evaluated with spanning tests, following the approach of (Kan and Zhou 2012).

The mean variance spanning test is developed, using the regression based model:

\[
R_{TA} = XB + E
\]

Where \(R_{TA}\) is a vector of returns of test asset if N=1 for time frames 1 to T. If N>1 then \(R_{TA}\) is a matrix \(T \times N\). \(X\) includes a set of benchmark returns \(K\) for time frames 1 to T.

\[
X = \begin{pmatrix}
1 & R_{1,1} & \cdots & R_{1,K} \\
\vdots & \vdots & \ddots & \vdots \\
1 & R_{T,1} & \cdots & R_{T,K}
\end{pmatrix}
\]

\(^2\) Mean-Variance spanning tests have widely used in literature to explore the benefit of diversification over several asset classes, e.g., (Belousova and Dorflteitner 2012) explored the effect of adding commodities on stock portfolios from the euro investor perspective.
B is K+1 dimensional coefficients vector $[\alpha, \beta]'$, where $\beta = (\beta_1, \cdots, \beta_K)$ and E is the error term vector $(\epsilon_1, \cdots, \epsilon_K)'$

The variance covariance matrix between test assets N and benchmark asset K can be rewritten as follow:

$$\Omega = \begin{pmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{pmatrix}$$

Where $V_{1,1}$ and $V_{2,2}$ are respectively the variance of the test and the benchmark assets and $V_{2,1}, V_{1,2}$ are the covariance returns between the two asset panels.

From (Huberman and Kandel, 1987), we can derive the null hypothesis for spanning as:

$$H_0: \alpha = 0, \delta = 1 - \beta 1 = 0$$

Under the null hypothesis, which is a joint test, adding the test asset to the benchmark assets will not improve statistically the return of the resulting optimal portfolio or reduce the risk. Geometrically, when $H_0$ holds the minimum variance frontiers of N assets and N+K assets coincide. Respectively, if $H_0$ is rejected, adding test assets will result in an improvement of return with respect to the benchmark asset portfolios. Consequently, the minimum variance frontier of N+K assets will shift towards risk contraction and/or return expansion.

Huberman and Kandel regression based approach tests the aforementioned null hypothesis in the regression model using the likelihood ratio test. Under the normality assumption, (Kan and Zhou 2008) rewrite the null hypothesis as:

$$H_0: \Theta = O_{2 \times N} \text{ since } \Theta = AB - C \text{ where}$$

$$A = \begin{bmatrix} 1 & 0_K \\ 0 & -1_K \end{bmatrix} \text{ and } C = \begin{bmatrix} 0_N \\ -1_N \end{bmatrix}$$

The test requires the definition of the following estimator matrices:

$$\hat{\Theta} = T A (X'X)^{-1} A' \text{ with } \hat{\Theta} = \Theta \Sigma^{-1} \Theta'$$
Where $\hat{\theta}$ is the maximum likelihood estimator of $\theta$ and $\hat{\Sigma}$ is the estimator of the covariance matrix of error terms. Denoting $\lambda_1$ and $\lambda_2$ as the two eigenvalues of $\hat{H}\hat{G}^{-1}$ where $\lambda_1 \geq \lambda_2 \geq 0$, the likelihood ratio can be expressed as:

$$LR = T \sum_{i=1}^{2} \ln (1 + \lambda_i)$$

The distribution of the likelihood ratio is asymptotically chi-squared with degrees of freedom equal to the number of restrictions under the null hypothesis.

Similar to the likelihood ratio, other asymptotic chi-squared tests can be employed, namely the Wald and Lagrange multiplier tests:

$$W = T(LR) = T(\lambda_1 + \lambda_2)$$

$$LM = T \sum_{i=1}^{2} \frac{\lambda_i}{1 + \lambda_i}$$

### 3.2 Step-down tests

It is important to notice that the presented regression based tests in the previous section, are a joint test for the spanning case of two minimum variance efficient frontiers derived from (1) all asset universe and (2) benchmark asset. It can be shown that the first part of the test $\alpha$ is related to the departure from the global minimum variance portfolios of the two efficient frontiers while $\delta$ measures the deviation from the tangent optimal portfolios. If the test rejects the spanning hypothesis at the traditional significance level of 5%, one concludes that the inclusion of the test asset helps to improve both optimal portfolios i.e.; global minimum variance and the tangent portfolios in the mean variance space. Statistically, the joint spanning tests discussed, overweight $\alpha$ than $\delta$ and thus do not allow distinguishing the source of rejection.

Based on (Anderson 1984), (Kan and Zhou 2012) proposed the step-down procedure for spanning test helping to (1) determine the source of rejection i.e., global minimum variance portfolio and/or tangent portfolio and to (2) allocate different significance levels of rejection.
between the two optimal portfolios in the test. The latter advantage circumvents the limits of hypothesis testing decisions at traditional significance levels while the economic results are different. An important difference in the tangent portfolios, while statistically is not significant, it can be economically important.

The implementation of step-down procedure is based on two $F$-test. The first $F_1$ tests $\alpha = 0$ and $F_2$ evaluates $\delta = 0$ but conditional on the constraint $\alpha = 0$.

4 Empirical Illustration

The empirical illustration starts with presenting the dataset and an overview of the descriptive statistics. In the next subsection, the joint spanning tests discussed in section 3 are performed based on efficient frontiers of benchmark assets derived from FB stocks and benchmark plus several combinations of FB stocks as test assets. For all optimal portfolios, the step-down procedure is applied to determine the source of departure from spanning hypothesis. All tests are performed on the dataset during three period: crisis period, post crisis and overall period.

4.1 The dataset

The dataset consists of daily prices of 57 stocks listed in Casablanca Stock Market. 12 stocks are identified as FB referring to criteria of governance and asset property. The dataset covers two periods. The first period represents the financial crisis time frame assessed by the stock market local authorities in Morocco from March 2008 to January 20093. The second period represents a bull market from February 2009 to January 2011.

4.2 Results

Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Crisis Period</th>
<th>Post Crisis Period</th>
</tr>
</thead>
</table>

3 During this period the market index Moroccan All Stock Index (MASI) plunged about 37%.
<table>
<thead>
<tr>
<th></th>
<th>FB(*)</th>
<th>NFB(**)</th>
<th>FB</th>
<th>NFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stocks</td>
<td>45</td>
<td>12</td>
<td>45</td>
<td>12</td>
</tr>
<tr>
<td>Average Return</td>
<td>-0,14%</td>
<td>-0,17%</td>
<td>0,07%</td>
<td>0,09%</td>
</tr>
<tr>
<td>Maximum Return</td>
<td>0,04%</td>
<td>0,10%</td>
<td>0,17%</td>
<td>0,33%</td>
</tr>
<tr>
<td>Minimum Return</td>
<td>-0,53%</td>
<td>-0,60%</td>
<td>-0,08%</td>
<td>-0,10%</td>
</tr>
<tr>
<td>Average Standard deviation</td>
<td>1,99%</td>
<td>2,29%</td>
<td>1,80%</td>
<td>1,85%</td>
</tr>
</tbody>
</table>

(*) Family Business

(**) Non Family Business

Table 1 presents some descriptive statistics. While during crisis period, on average FB stocks seems to perform better in terms of return and risk than non-family stocks however the latter exhibit more return during post crisis period. FB stocks present in both periods less volatility than their counterparts.

Figure 1: Initial Stocks in the Mean-Variance space

Figure 1 presents positions of initial stocks in Mean-Variance space during overall period. Black circle points represent non-family stocks and red points show the position of FB
stocks. Visually, one may notice that FB stocks do not exhibit any superior feature from one or both specification Return and/or risk in the overall periods.

The minimum efficient frontier consisting of non family stocks is generated using the P1. Figures 2a and 2b present the geometric representation of the minimum variance frontiers derived from NFB solely during the crisis and post crisis period. One may notice the shift upward of the efficient frontier resulting from the recovery period in return dimension after the crisis.

Figure 2a: Minimum variance frontier of NFB stocks (crisis period)  
Figure 2b: Minimum variance frontier of NFB stocks (post crisis period)

To explore the benefit of diversification over FB stocks, a second efficient frontier is generated using all dataset i.e, FB and NFB stocks. Figures 3a en 3b presents the two efficient frontiers during the crisis and the post crisis periods. The gray frontier represents all optimal portfolios derived only from NFB stocks while the dashed curve shows all optimal portfolios where FB stocks are included with different weights.

Contrasting the frontiers, whether in the crisis or after the crisis period, visually suggests an added value of FB in optimal portfolios. In the crisis period, it is noticeable that the inclusion
of FB results in a shift of the efficient frontier to the left side in the direction of risk contraction up to the return level of 0.072%. After the crisis, FB stocks help to improve both risk and return of non family optimal portfolios. This improvement is more pronounced for higher level of returns.

Figure 3a: Diversification impact of FB stocks (crisis period)  
Figure 3b: Diversification impact of FB stocks (post crisis period)

The diversification over FB stocks effect is tested by employing the three spanning joint tests presented in section 3: LR, W and LM. The 12 FB stocks are specified as test assets while 45 NFB are presented as benchmark assets.

The table 2 presents test statistics and the respective p-values for the null hypothesis that FB stocks span optimal portfolios consisting of NFB assets. The three joint tests reject the spanning hypothesis at significance level of 5% and suggest a departure in both global minimum variance and tangent portfolios. This result confirms the geometric interpretation of efficient frontiers in figure 2a and 2b and hence prove the benefit of diversification over FB stocks in bull and bear market.
Table 2: Mean Variance spanning tests

<table>
<thead>
<tr>
<th></th>
<th>Wald</th>
<th>LR</th>
<th>LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>p-Value</td>
<td>Test</td>
</tr>
<tr>
<td>Overall Period</td>
<td>270.439</td>
<td>0.000</td>
<td>231.699</td>
</tr>
<tr>
<td>Crisis Period</td>
<td>122.794</td>
<td>0.000</td>
<td>98.669</td>
</tr>
<tr>
<td>Post Crisis Period</td>
<td>199.806</td>
<td>0.000</td>
<td>171.229</td>
</tr>
</tbody>
</table>

To explore the source of departure and the specific family stocks contributing in the diversification benefit the down-step procedure is implemented. The F tests are computed using non-family stocks as benchmark assets and 13 tests assets representing respectively individual FB stocks from 1 to 12 reported in table 3 with their names and the last test asset represents a portfolio consisting of all family stocks.
<table>
<thead>
<tr>
<th>Stock</th>
<th>Overall Period</th>
<th>Crisis Period</th>
<th>Post Crisis Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F-test</td>
<td>p-value</td>
<td>F-test</td>
</tr>
<tr>
<td>1 DARI COUSPAT</td>
<td>13.424</td>
<td>0.000</td>
<td>12.748</td>
</tr>
<tr>
<td>2 CARTIER SAADA</td>
<td>14.322</td>
<td>0.000</td>
<td>14.172</td>
</tr>
<tr>
<td>3 AUTO NEIMA</td>
<td>25.291</td>
<td>0.000</td>
<td>18.143</td>
</tr>
<tr>
<td>4 UNIMER</td>
<td>11.805</td>
<td>0.000</td>
<td>9.101</td>
</tr>
<tr>
<td>5 AFRIQUIA GAZ</td>
<td>12.113</td>
<td>0.000</td>
<td>11.554</td>
</tr>
<tr>
<td>6 COLORADO</td>
<td>0.221</td>
<td>0.000</td>
<td>0.264</td>
</tr>
<tr>
<td>7 CTM</td>
<td>0.151</td>
<td>0.000</td>
<td>0.165</td>
</tr>
<tr>
<td>8 ALUM MAROC</td>
<td>9.908</td>
<td>0.000</td>
<td>5.325</td>
</tr>
<tr>
<td>9 OULMES</td>
<td>11.877</td>
<td>0.000</td>
<td>5.325</td>
</tr>
<tr>
<td>10 SOTHENA</td>
<td>14.872</td>
<td>0.000</td>
<td>11.554</td>
</tr>
<tr>
<td>11 SNEP</td>
<td>5.330</td>
<td>0.005</td>
<td>3.144</td>
</tr>
<tr>
<td>12 ATLANTA</td>
<td>5.283</td>
<td>0.005</td>
<td>2.057</td>
</tr>
<tr>
<td>13 All</td>
<td>9.644</td>
<td>0.000</td>
<td>3.292</td>
</tr>
</tbody>
</table>

Table 3: Mean-Variance Spanning Tests
The table 3 presents two sets of mean-variance spanning tests. The first test is an F-test of $H_0: \alpha = 0$ and $\delta = 0$. The second test is a step down test where $F_1$ is an F-test of $\alpha = 0$ and $F_2$ is an F-test of $\delta = 0$ conditional on $\alpha = 0$. The results are presented for the overall period as well as for crisis and post crisis periods.

From the joint test, the spanning hypothesis is rejected for all FB stocks for both sub periods and overall period at the traditional significance level 5%. The corresponding $p$-value are very small. However, for individual FB, one may remark that not all of them have an added value with respect to the benchmark assets in the mean variance dimensions. For example, in the overall period, the joint test for $H_0: \alpha = 0$ and $\delta = 0$, family stocks COLORADO and CTM are statistically insignificant and hence the efficient frontier of NFB portfolios and augmented FB with COLORADO and/or CTM efficient frontiers coincide. In the crisis period, the diversification over 7 FB stocks have an impact in the risk reduction and/or the return magnification. The diversification over 9 FB stocks during the bull market period allows shifting the benchmark asset frontier in the direction of risk and/or return improvements. To explore the source of spanning rejection table 3 reports the F-test of step-down procedure. $p$-value of $F_1$ relative to an added value with respect to tangent portfolio suggests that there is no statistical evidence of any impact except for one FB stock in the overall period SNEP. However, the diversification over large part of FB stocks shows a strong departure in the global minimum variance portfolios. There are 10 over 13 in overall period, 9 in the crisis period and 10 in the post crisis period. This finding suggests that the family stocks in the portfolio contribute mainly in the risk reduction.
5 Conclusion

Starting from the seminal work of (Markowitz 1952), a bulk of literature explored the diversification effect over several asset classes in stock portfolios: bonds, commodities, futures, options.. etc,. This contribution focuses on the added value of FB stocks when the latter are considered in stock portfolios in the mean variance dimensions in both bull and bear market. Contrasting the geometric representation of minimum variance frontiers of portfolios consisting only of NFB stocks and augmented with family business stocks, suggests a visual deviation in both dimensions.

The geometric analysis is supplemented by asymptotic joint spanning tests: likelihood ratio, wald and lagrange Multiplier test. The regression based tests confirms statistically the added value of NFB in stock portfolios.

To explore the source of deviation, the step-down procedure is implemented. Results reveal that large part of family business stocks helps reducing risk solely and there is no a statistical evidence at the traditional significance level of 5% on their contribution in the tangency portfolio whether in crisis or bull market. This result suggest that family business stocks are good investment when considering risk reduction strategies whatever the market conditions.
References


